PROJECTED WRITTEN NOTES FROM THE M325K LECTURE ON TUESDAY, JANUARY 23, 2024, on
The CONDITIONAL (IF, THEN) STATEMENT AND RELATED STATEMENTS and a Congruence (mod n) Theorem

CLASS #3

The Definition of "$a \equiv b \pmod{n}$".

For integers a, b, and n, such that $n > 0$,
"$a \equiv b \pmod{n}$" if and only if
there exists an integer k such that $(a-b) = nk$.
(i.e. $(a-b) = nk$ for some integer k.)

The Conditional Statement "If p, then q"

Also written: "$p \to q$"

Ex: "If John lives in Austin, Then John lives in Texas."

Assume that x is a real #: If $x \neq 0$, Then $x^2 > 0$.

The Truth Table for "$p \to q$":

p	q	$p \to q$	$\sim p \vee q$	$p \wedge \sim q$
T	T	T	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

$p \wedge \sim q \equiv \sim(p \to q)$

$\equiv$

Every conditional (IF, THEN) is equivalent to an OR statement: $p \Rightarrow q \equiv \sim p \vee q$.

Ex: If $\underbrace{\text{Bill pays his rent}}_{p}$, then $\underbrace{\text{he can stay another month}}_{q}$.

$$(p \to q) \equiv \sim p \vee q$$

Bill doesn't his rent OR (if he does) he can stay another month.

The negation of "If p, Then q" is "p AND NOT q"

$$\sim(p \to q) \equiv (p \wedge \sim q)$$

The Negation of "If Bill pays his rent, Then he can stay another month"

is

"Bill paid his rent, but (and) he could not stay another month".

Related Conditionals

Given a particular conditional statement
$p \to q$ (The ORIGINAL COND'L),

There are three (3) other conditional statements related to the Original $p \to q$.

Its Converse: $q \to p$

Its Inverse: $\sim p \to \sim q$

Its Contrapositive: $\sim q \to \sim p$.

$(p \to q) \not\equiv (q \to p)$, but $(p \to q) \equiv (\sim q \to \sim p)$,
that is, An Original Conditional Statement IS EQUIVALENT TO its Contrapositive!

p	q	$p \to q$	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim q \to \sim p$	$p \wedge \sim q$
T	T	T	T	F	F	T	T	F
T	F	F	T	F	T	T	F	T
F	T	T	F	T	F	F	T	F
F	F	T	T	T	T	T	T	F
		The ORIGINAL Conditional	The Converse			The Inverse	The Contra-Positive	The Negation of $p \to q$

The Tables show that:

(1) $p \to q \equiv \sim q \to \sim p$, (2) $q \to p \equiv \sim p \to \sim q$, (3) The Negation has NO "IF"!

(3) $\sim(p \to q) \equiv$ p AND NOT q.

Related Conditionals:

Ex: The Original Conditional: $(p \to q)$

"If John lives in Austin, then John lives in Texas".

Its: Converse : $(q \to p)$:

"If John lives in Texas, then John lives in Austin.

Its Inverse : $(\sim p \to \sim q)$

"If John doesn't live in Austin, then John doesn't live in Texas"

Its Contrapositive : $(\sim q \to \sim p)$

"If John doesn't live in Texas, Then John doesn't live in Austin."

FACT: The Contrapositive is Equivalent to The Original conditional

$$(\sim q \to \sim p) \equiv (p \to q)$$

Other Ways to state "IF p, THEN q" ($p \to q$) in words

"If p, Then q" $\equiv$ "q, If p." $\equiv$ "p Implies q"

$\equiv$ "p is a sufficient condition for q".

$\equiv$ "q is a necessary condition for p"

why this? $\to$

$\equiv$ "Without q, you don't have p"

$\equiv \sim q \to \sim p$ (The Contrapositive of $p \to q$)

$\equiv p \to q$

The Connectives so far:

AND	$p \wedge q$
OR	$p \vee q$
Implies	$p \to q \equiv$ q, IF p.

A New Connective : "Only if" as in "t, ONLY IF s".

When determining the interpretation, understand that
The Statement DIRECTLY FOLLOWING ~~the~~ "ONLY IF"
is a Necessary Condition for The other statement.

Give an equivalent Wording for the following:

"Only if $\underbrace{\text{I work hard}}_{s}$ is $\underbrace{\text{my Boss happy}}_{t}$."

$\equiv$ "s is necessary for t"

$\equiv$ "$\sim s \to \sim t$"

$\equiv t \to s$

$\equiv$ "If my Boss is happy, Then I work hard."

$17 \div 3$

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ -15 \\ \hline 2 = r \end{array}$$

$17 = 5 \times 3 + 2$

$17 - 2 = 5 \times 3$

$a - b = q \cdot n$

$17 \equiv (2) \pmod 3$

Consider the modulus 8, $n = 8$:

FIND 4 integers greater than 21 and 4 integers less than 21 which are congruent to 21 modulo 8.

"congruent (mod 8) to 21"

$$21 \xrightarrow{+8} 29 \xrightarrow{+8} 37 \xrightarrow{+8} 45 \xrightarrow{+8} 53$$

$\equiv \quad \equiv \quad \equiv \quad \equiv \pmod 8$

$$-11 \xleftarrow{-8} -3 \xleftarrow{-8} 5 \xleftarrow{-8} 13 \xleftarrow{-8} 21$$

$\equiv \quad \equiv \quad \equiv \quad \equiv. \pmod 8$

~~8~~

$-11 \equiv 21 \pmod 8$? ✓

$-11 - 21 = -32 = 8(-4)$.